

Massively Parallel Evolutionary Computation for Empowering Electoral Reform

Quantifying Gerrymandering via Multi-objective Optimization and Statistical Analysis

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Abstract—Important insights into redistricting can be gained by formulating and analyzing the problem within a large-scale optimization framework. Redistricting is an application of the set-partitioning problem that is *NP*-Hard. We design a hybrid metaheuristic as the base search algorithm. With our grant on the Blue Waters supercomputer, we extend our algorithm to the high-performance-computing realm by using MPI to implement an asynchronous inter-process communication framework. We experimentally demonstrate the effectiveness of our algorithm to utilize thousands of processors for the congressional redistricting problem in the state of Wisconsin. The massive computing power allows us to extract new substantive insights that closely mesh with the framework that the Supreme Court has elucidated for electoral reform.

I. INTRODUCTION

Partisan gerrymandering is unconstitutional, but the Court is unsure of how to identify a partisan gerrymander. Justice Souter writes that “some intent to gain political advantage is inescapable whenever political bodies devise a district plan, and some effect results from the intent... [T]he issue is one of how much is too much.” Scalia reiterates, “[t]he central problem is determining when political gerrymandering has gone too far.” Kennedy identifies a key problem, that “[n]o substantive definition of fairness in districting seems to command general assent.” Plainly, if we do not know how to measure partisan fairness, we cannot determine how much political effect is too much.

In *Vieth v. Jubilerer*, the Court commented that developing a manageable standard for judging partisan gerrymandering is sufficiently difficult that partisan gerrymandering is not justiciable. Four justices concurred on the question of justiciability—one justice shy of a majority—leaving partisan gerrymandering claims within the purview of the Court and leaving open the possibility that an eventual workable standard will one day be identified and accepted by the Court.

We take the Court’s legal framework and place it within the proper computational framework. We aim to fulfill Justice Kennedy’s words in *Vieth*: “Technology is both a threat and a promise. On the one hand, if courts refuse to entertain any claims of partisan gerrymandering, the temptation to use

partisan favoritism in districting in an unconstitutional manner will grow. On the other hand, these new technologies may produce new methods of analysis that make more evident the precise nature of the burdens gerrymanders impose on the representational rights of voters and parties. That would facilitate court efforts to identify and remedy the burdens, with judicial intervention limited by the derived standards.”

II. PROBLEM FORMULATION

Redistricting is an application of the set partitioning problem that is *NP*-Hard. The goal of the redistricting problem is to identify a plan that optimizes objectives while satisfying legal constraints. We have a set of N geographic units, $u_1, u_2, \dots, u_N$, that we wish to partition into a set of K districts, $d_1, d_2, \dots, d_K$.

The population of the N units is denoted by $p_1, p_2, \dots, p_N$. So, if the districts are equipopulous, then the population in each district would be the average population, $\bar{P}$, given by

$$\bar{P} = \frac{1}{K} \sum_{i=1}^N p_i.$$

Let $\mathbf{X}$ be an $N \times K$ matrix with elements, x_{ik} , denoting our decision variables. To specify a map, these variables are chosen for $1 \leq i \leq N$ and $1 \leq k \leq K$ so that

$$x_{ik} = \begin{cases} 1 & \text{if unit } u_i \text{ is assigned to district } d_k \\ 0 & \text{otherwise.} \end{cases}$$

The population in district k is then

$$P_k = \sum_{i=1}^N x_{ik} p_i \quad \text{for } k = 1, 2, \dots, K.$$

Constraints. We have constraints of at least three types.

- 1) Each unit must be assigned to exactly one district,

$$\sum_{k=1}^K x_{ik} = 1 \quad \text{for } i = 1, 2, \dots, N.$$

- 2) The maximum population deviation across all K districts is no greater than a specified value M . That is, for any two districts, d_i and d_j ,

$$|P_{d_i} - P_{d_j}| \leq M \quad \text{for } i, j = 1, 2, \dots, K.$$

- 3) The units in each district must form a connected set.

Other constraints like compactness or preserving political subdivisions may exist.

We use a parallel evolutionary algorithm to identify a very large set of high quality independent maps that are representative of possible redistricting maps. Such a set would serve as a baseline for assessing whether a disputed map is constitutionally permissible.

III. Wisconsin

The Court is set to hear *Whitford v. Gill* this Fall. Justice Ginsburg recently said it might be the “most important” case they will hear this next term. In 2012, the Republicans received 48.6% of the statewide vote yet triumphed in 60 of the 99 state assembly seats. This seems unfair on its face, but satisfying the legal criteria for an unconstitutional partisan gerrymander is not simple.

Several methods have been proposed for measuring. Recently, some of these methods have focused on simulating electoral maps [1]–[4]. Some of these methods implement Markov Chain Monte Carlo (MCMC) algorithms that are intended to sample from a probability distribution based on a constructing a Markov chain whose distribution is the stationary distribution underlying the data. MCMC is theoretically attractive but not computationally tractable for redistricting because the redistricting problem size is prohibitively large. Current MCMC implementations simplify the problem to examine only changes that are at most 5-10% from the current district. This is obviously problematic if the current district is itself a gerrymander. Plainly we are interested not only in a theoretically appropriate algorithm, but one that does not require redefining the problem so that it is computationally feasible. MCMC breaks down on this latter criterion.

Our project proposes a method that is both based on an evolutionary algorithm that is scalable on massively parallel computing architecture as well as fitting in with the Court’s articulated framework for a manageable partisan gerrymandering standard. While MCMC implementations have sometimes taken 72 hours to identify only hundreds of maps [4], our implementation has effectively identified millions of maps by utilizing 4096 processor cores. The computational scalability was achieved using asynchronous inter-process communications and non-blocking MPI calls. Figures 1 and 2 show some of the range in plans that are possible when one is optimizing more heavily on one objective than another.

IV. CONCLUSION

One reason that the Supreme Court has been unable to identify a standard for partisan gerrymandering is because they have unwittingly forwarded a standard that is not only computational in nature but whose computational complexity

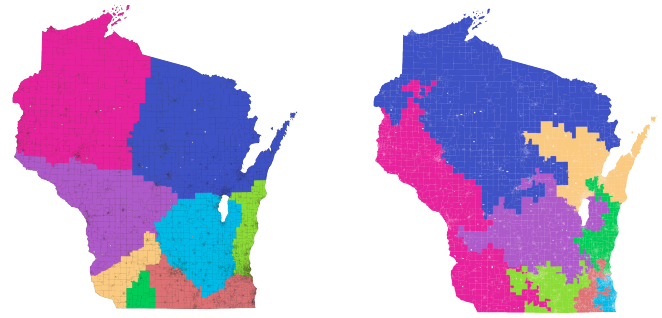


Fig. 1. Compact map

Best Fitness map

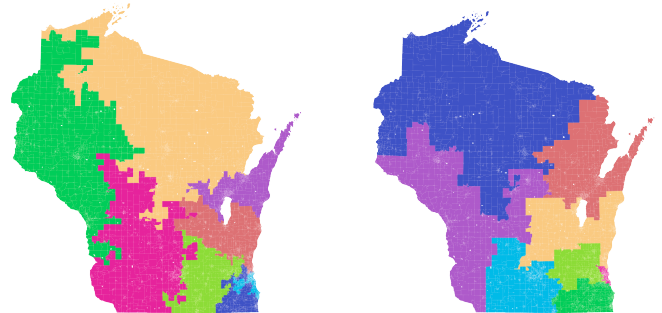


Fig. 2. Democratic map

Republican map

is immense. In this sense, electoral reform for redistricting requires harnessing massive computing power. We have presented a high-performance computing solution that enables the analysis that Supreme Court requires.

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