Massively Parallel Evolutionary Computation for Empowering Electoral Reform

Quantifying Gerrymandering via Multi-objective Optimization and Statistical Analysis

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Summary

Political redistricting is formulated as a large-scale discrete combinatorial optimization problem.
Implementation Features:

- Scalable Evolutionary Algorithm
- Multi-Objective Optimization
- Massively Parallel Implementation
- Spatially Cognizant Evolutionary Operators
- Tightly coupled with Supreme Court mandates

Description

We develop an massively scalable parallel evolutionary computational redistricting algorithm. Our parallelization of the algorithm harnesses parallel computing power provided by supercomputers via the coupling of Evolutionary Algorithm (EA) search processes and a highly scalable message passing model that maximizes the overlapping of computing and communication at runtime. The efficiency of the EA operators is enhanced by a spatially cognizant design that minimizes space traversal of infeasible portions of the solution space. Experimental results demonstrate desirable effectiveness and scalability (up to 131,072 processors) for solving large redistricting problems, enabling research into the relationship between democratic ideals and phenomena such as gerrymandering.

We illustrate with an application to congressional redistricting in Wisconsin, where we generate more than 2 million feasible maps using 4,096 processors on the Blue Waters supercomputer. Our application demonstrates the feasibility of this type of analysis on a computational, statistical, and substantive realm.

Problem Formulation

Partition N geographic units, $u_1, u_2, \ldots, u_N$, into K districts, $d_1, d_2, \ldots, d_K$.

$$x_{ik} = \begin{cases} 1 & \text{if unit } u_i \text{ is assigned to district } d_k \\ 0 & \text{otherwise.} \end{cases}$$

Multi-objective optimization encompassing legal requirements and balancing legal constraints.

Legal Requirements:

- Equi-populous districts: $\overline{P} = \frac{1}{K}\sum_{i=1}^{N} p_i$
- Contiguity
- Protection of Minority Voting Rights

An Array of Possible Legal Constraints:

- Compactness: $C_{IPQ} = \frac{4\pi A}{P^2}$
- Traditional Districting Principles
- Partisan Metrics (e.g. competitiveness): $T_p(1 + \alpha T_e)\beta$, where

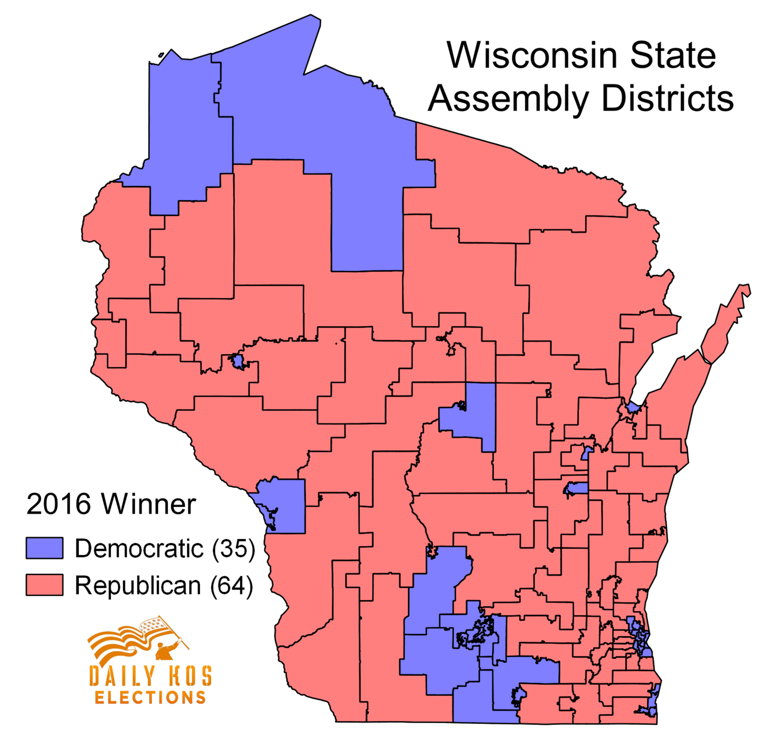
$$T_p = \frac{1}{K}\left(\sum_{k=1}^{K} \left| \frac{R_k}{D_k + R_k} - \frac{1}{2} \right|\right), \text{ and } T_e = \left| \frac{B_R}{K} - \frac{1}{2} \right|$$

for $0 \le T_p \le 0.5$, $0 \le T_e \le 0.5$, and $\alpha, \beta \ge 0$

where T_p is a vote deviation from 0.50,
T_e is a weighting factor for seat differential,
B_R is the number of seats favoring Republicans,
α is a weight for T_e, and
β a normalizing constant

Whitford v. Gill

- In 2012, Republicans won 48.6% of the vote and 60 of the 99 State Assembly seats.
- The Supreme Court has not agreed on a manageable standard for adjudicating partisan gerrymandering cases.
- Our model is a computational solution that fits within the Court's legal framework and provides a path forward to the illusive standard that has vexed the Courts for more than 5 decades.



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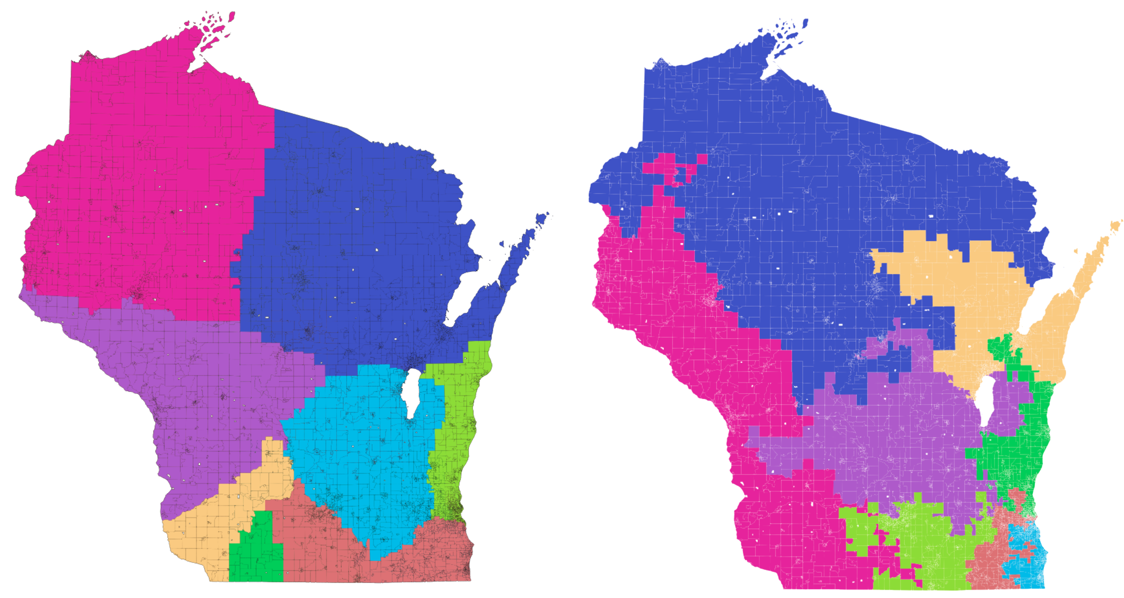


Figure 1: Compact map Best Fitness map

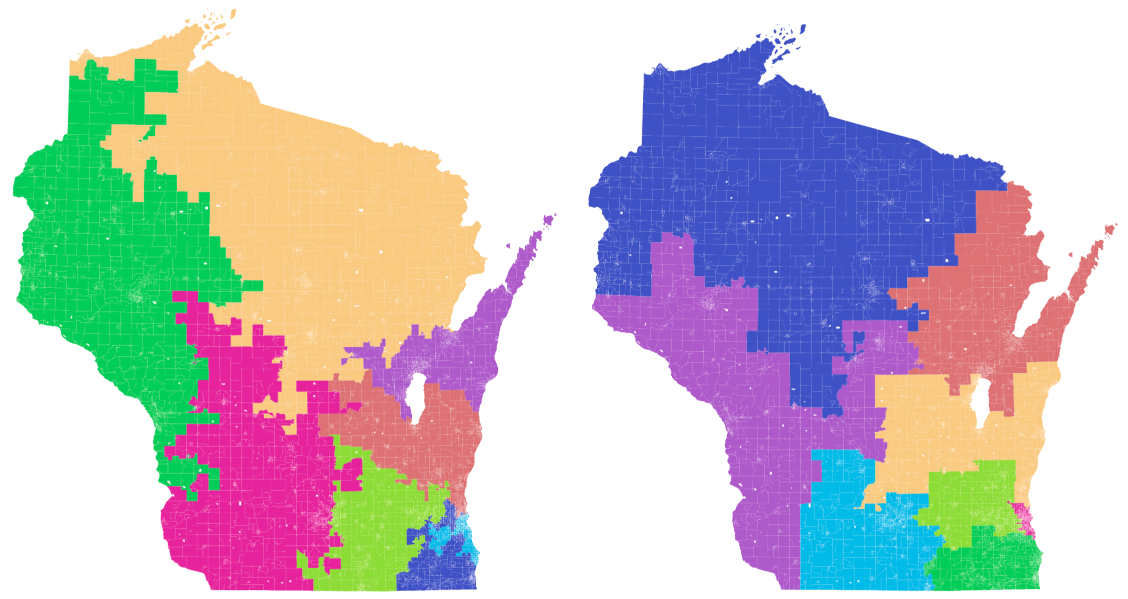
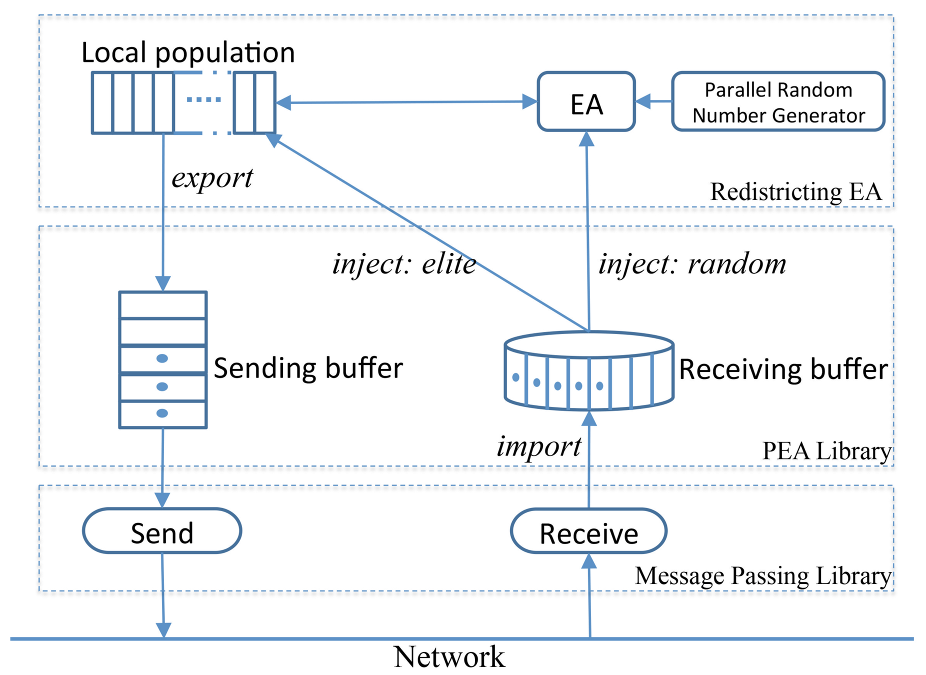


Figure 2: Democratic map Republican map

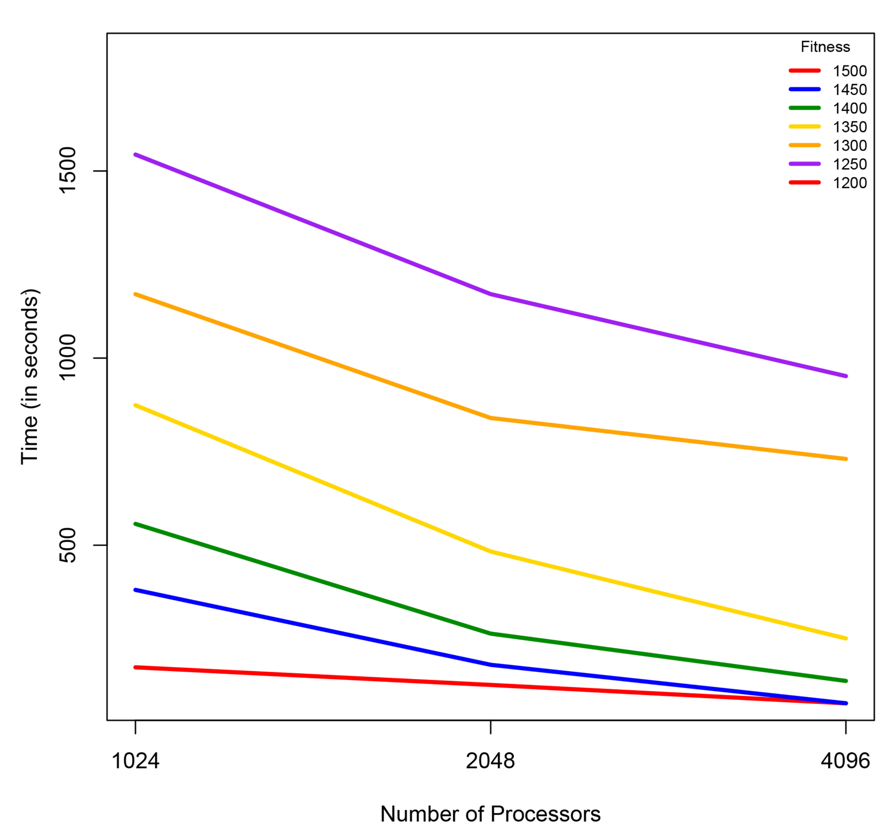
Justice Anthony Kennedy (*Vieth v. Jubelirer*)

"Technology is both a threat and a promise... these new technologies may produce new methods of analysis that make more evident the precise nature of the burdens gerrymanders impose on the representational rights of voters and parties."

Asynchronous Interprocess Communication

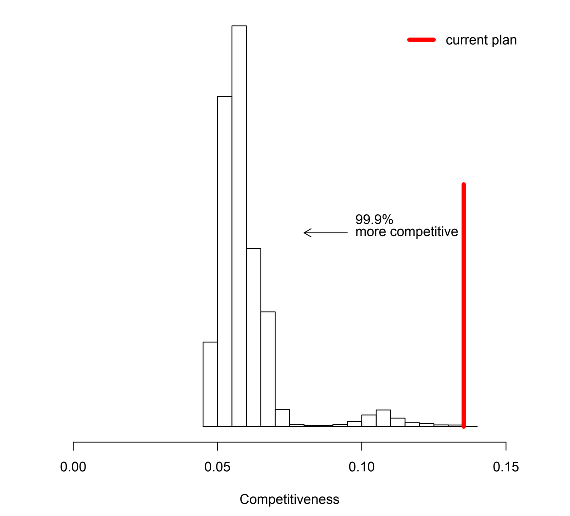


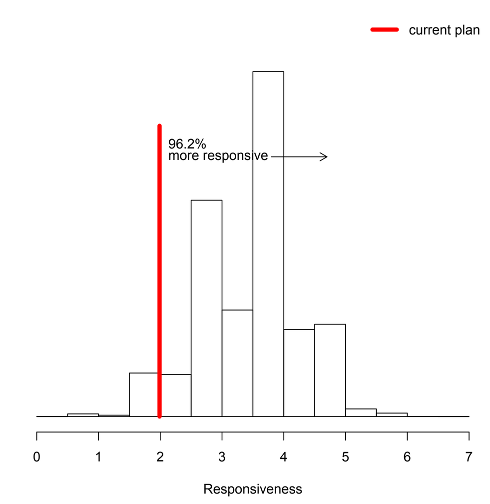
Scalability

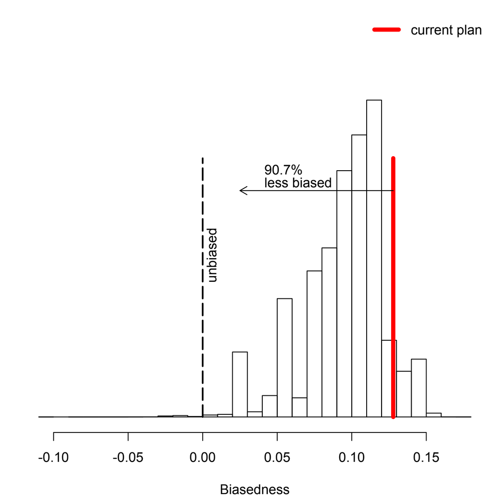


Extreme Outlier Statistical Analysis

Comparison of existing map to 2.4 million other feasible maps on a set of partisan metrics.







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